Heat transport experiments on quantum nanostructures

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Outline:
1. Heat management in nanostructures
2. Measurement of heat currents and thermal conductance; analogy between “electronics” and “heattronics”
3. Basic heat transport mechanisms
4. Recent and ongoing experiments
5. Quantum heat conductance
6. Quantum refrigerators, quantum heat switch
Main topics

Energy relaxation

(Quantum) heat conductance

Thermometry

Quantum refrigerators

Measuring heat transport
Generic thermal model for an electronic conductor
Analogy between "electronics" and "heattronics"

\[ P = C\dot{T} + G_{th}(T - T_{bath}) \]

\[ I = CV\dot{V} + G(V - V_0) \]
Measurement of currents

Electric currents can be measured by observing the magnetic field.

Heat current $P$ through $G_{th}$? Measure the Joule power $IV$, and use continuity equation.
Thermal response of an absorber

Steady-state heating ("bolometer")

Response to a heat pulse ("calorimeter")
Temperature in an electronic device

\[ f(E) = \frac{1}{1 + e^{(E-\mu)/k_B T}} \]
The energy distribution of electrons in a small metal conductor

The distribution is determined by energy relaxation:

**Equilibrium** – Thermometer measures the temperature of the "bath"

**Quasi-equilibrium** – Thermometer measures the temperature of the electron system which can be different from that of the "bath"

**Non-equilibrium** – There is no well defined temperature measured by the "thermometer"

Illustration: diffusive normal metal wire
H. Pothier et al. 1997
For normal conductors (NIN junction), density of states (DOS) is almost constant over the small energy interval:

\[
I(V) = eT^2 \int [f_1(E - eV) - f_2(E)]dE
\]

Quite generally:

\[
\int [f_1(E - eV) - f_2(E)]dE = eV
\]

Ohmic, no temperature dependence
Current through an ideal NIS-tunnel junction with low-transparency

\[ I = \frac{1}{eR_T} \int n_S(E)[f_N(E - eV) - f_S(E)]dE \]
Current through an ideal NIS-tunnel junction with low-transparency

\[
I = \frac{1}{eR_T} \int n_S(E)[f_N(E - eV) - f_S(E)]dE \\
= \frac{1}{2eR_T} \int n_S(E)[f_N(E - eV) - f_N(E + eV)]dE
\]
NIS-thermometry

\[ I = \frac{1}{2eR_T} \int n_S(E)[f_N(E - eV) - f_N(E + eV)]dE \]

Probes electron temperature of N island (and not of S!)

Graphs showing I vs. V and V vs. T for different temperatures.
NIS-thermometry at mK temperatures


Potential to operate down to 1 mK
Energy current in a tunnel junction

Energy current (from conductor 1)

\[ P(V) = \frac{1}{e^2 R_T} \int (E - eV) n_1(E - eV) n_2(E)[f_1(E - eV) - f_2(E)]dE \]

Compare to:

\[ I(V) = \frac{1}{e^2 R_T} \int eN_1(E - eV) N_2(E)[f_1(E - eV) - f_2(E)]dE \]

For a NIN junction (constant DOSes)

\[ P(V) = \frac{1}{e^2 R_T} \int (E - eV)[f(E - eV) - f(E)]dE = -\frac{V^2}{2R_T} = -IV/2 \]

The Joule power is distributed equally between 1 and 2 in this case.
Electronic NIS-coolers

Cooling power of a NIS junction:

\[ P_{\text{NIS}} = \frac{1}{e^2 R_T} \int dE (E - eV) n_S(E) \left[ f_N(E - eV) - f_S(E) \right] \]

Optimum cooling power is reached at \( V \equiv \Delta/e \):

\[ P_{\text{NIS}} \approx 0.6 \frac{\Delta^2}{e^2 R_T} \left( \frac{k_B T_N}{\Delta} \right)^{3/2} \]

Efficiency (coefficient of performance) of a NIS junction cooler:

\[ \eta \approx \frac{k_B T}{\Delta} \]

For reviews, see Giazotto et al., Rev. Mod. Phys. 78, 217 (2006); Muhonen et al., Reports on Progress in Physics 75, 046501 (2012).
Electron-phonon relaxation in metals at low $T$

Hot-electron effects in metals

F. C. Wellstood,* C. Urbina, † and John Clarke
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and Materials Sciences Division, Lawrence Berkeley Laboratory, Berkeley, California 94720
(Received 21 July 1993)

\[ \dot{Q}_{ep} = \Sigma \Omega (T_c^5 - T_p^5) \]

FIG. 2. Emission and absorption of phonons of wave vector $q$ by an electron of wave vector $k$. 

*现在的地址：
**过去的地址：
Measurement of electron-phonon coupling of a normal metal wire

K. L. Viisanen and JP, arXiv:1606.02985

Copper and silver thin film wires measured
$G_{th}$ - electron-phonon coupling

\[ \dot{Q} = \Sigma V (T_e^5 - T_p^5) \]

\[ G_{th} = 5 \Sigma VT^4 \]
Quasiparticle recombination

Superconducting gap $2\Delta$\[\text{Recombination with } 2\Delta \text{ phonon emission}\]

\[E_{ph} = 2\Delta\]

\[\frac{1}{\tau_{rec}} = \frac{1}{\tau_0} \sqrt{\pi} \left(\frac{2\Delta}{kT_c}\right)^{5/2} \sqrt{\frac{T}{T_c}} e^{-\frac{\Delta}{kT}}\]

Kaplan et al, 1976
Barends et al., 2008

This process represents electron-phonon relaxation in a superconductor at low $T$. The corresponding heat current is suppressed exponentially.

\[P_{qp-ph} \approx \frac{64}{63\zeta(5)} \Sigma \sqrt{V} T^5 e^{-\Delta/k_B T}\]
Measurement of recombination in a superconductor


$\tau^{-1} = 16 \text{ kHz for a single qp pair in Al}$
Electronic heat conduction

1D heat diffusion along x-axis of a uniform wire with cross-sectional area $A$

$$\dot{Q} = -G_{\text{th}} A \frac{dT}{dx}$$

In a metal, diffusive heat transport is governed by the Wiedemann-Franz law:

$$G_{\text{th}}^N = \mathcal{L}_0 G_N T$$

$$\mathcal{L}_0 = \pi^2 \left( \frac{k_B}{e} \right)^2 / 3$$

is the Lorenz number and $G_N$ is the electrical conductivity.
Quasiparticle heat conduction in a superconductor

Bardeen et al. 1958

\[ \gamma(T) = \frac{G_{th}}{G_{th}^N} = \frac{3}{2\pi^2} \int_{\Delta/k_B T}^\infty dx \frac{x^2}{\sech^2(x/2)} \approx \frac{3}{2\pi^2} (8 + 8a + 4a^2) e^{-a} \]

\[ a = \frac{\Delta}{k_B T} \]

Heat transport is exponentially suppressed at low temperatures in a superconductor!

\[ \frac{\Delta T_2}{\Delta T_1} = \frac{G_{th}}{G_{th} + G_{ep,2}} \]

(for small temperature differences)

J. Peltonen et al., PRL 2010
Recent measurement on Nb

A. Feshchenko et al. arXiv:1609.06519
Steady-state heat transport measurement through a single-electron transistor

B. Dutta et al., in preparation
Quantized (heat) conductance

Electrons:

Electrical conductance in a ballistic contact:

\[ \sigma_Q = \frac{2e^2}{h} \]

Thermal conductance:

\[ G_Q = \frac{\pi k_B^2}{6h} T \]

\( G_Q \) and \( \sigma_Q \) related by Wiedemann-Franz law

More generally:

Example of quantized thermal conductance: phonons in a nanobridge

Electromagnetic transfer of heat (photons)

Electron system

Lattice

Electrical environment

Schmidt et al., PRL 93, 045901 (2004)
Ojanen et al., PRB 76, 073414 (2007), PRL 100, 155902 (2008)
D. Segal, PRL 100, 105901 (2008)
L. Pascal et al., PRB 83, 125113 (2011)
Radiative heat transport in an electrical circuit

Voltage noise of a resistor:

\[ S_{V_i}(\omega) = 4\hbar \omega R_i n_i(\omega) \]

Bose distribution:

\[ n_i(\omega) = \frac{1}{e^{\hbar \omega / k_B T_i} - 1} \]

Current noise created by resistor 1:

\[ S_{I_1}(\omega) = S_{V_1}(\omega) / |Z_{tot}|^2 \]

\[ Z_{tot} = R_1 + R_2 \]

Spectrum of dissipation of energy created by resistor 1 and absorbed by resistor 2:

\[ S_{P_{12}}(\omega) = R_2 S_{I_1}(\omega) \]
Heat transported between two resistors

\[ P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \left[ S_{P12}(\omega) - S_{P21}(\omega) \right] = r \frac{\pi k_B^2}{12\hbar} (T_1^2 - T_2^2) \]

Radiative contribution to net heat flow between electrons of 1 and 2:

Coupling constant:

\[ r \equiv \frac{4R_1R_2}{(R_1 + R_2)^2} \]

Linearized expression for small temperature difference \( \Delta T = T_1 - T_2 \):

\[ P_\nu = r G_Q \Delta T \]

\[ G_Q = \frac{\pi k_B^2}{6\hbar} T \]
Classical or quantum heat transport?

\[ P_\nu = \int_0^\infty \frac{d\omega}{2\pi} \frac{4R_1 R_2 \hbar \omega}{|Z_t(\omega)|^2} \left( \frac{1}{e^{\hbar \omega/k_B T_1} - 1} - \frac{1}{e^{\hbar \omega/k_B T_2} - 1} \right) \]

\[ \frac{4R_1 R_2}{|Z_t(\omega)|^2} \]

\[ \omega_c \quad k_B T/\hbar \]

"Classical"

\[ G_\nu \sim r k_B \omega_C \]

"Quantum"

\[ G_\nu = r G_Q \]
Classical or quantum heat transport?

Classical:
\[
\frac{\hbar}{k_B T} \frac{1}{RC} \ll 1 \quad \frac{\hbar}{k_B T} \frac{R}{L} \ll 1 \quad \text{Johnson, Nyquist 1928}
\]

Quantum limited:
\[
\frac{\hbar}{k_B T} \frac{1}{RC} \gg 1 \quad \frac{\hbar}{k_B T} \frac{R}{L} \gg 1
\]

\( T = 300 \text{ K}, \ell = 1 \text{ cm}: \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} < 10^{-3} \ll 1 \)

\( T = 100 \text{ mK}, \ell = 100 \mu\text{m}: \frac{\hbar}{k_B T} \frac{1}{RC} \sim \frac{\hbar}{k_B T} \frac{R}{L} \sim 10^2 > 1 \)
Demonstration of photonic heat conduction

Tunable impedance matching using DC-SQUIDs

\[ L_J \Phi = \frac{\hbar}{2eI_{C,0}|\cos(\pi \Phi / \Phi_0)|} \]

Thermal model

2nd experiment

SAMPLE A in a loop ("matched")
[SAMPLE B without loop ("not matched")]

Heat transport in different set-ups

Loop geometry (Sample A)

\[ P^A_{\nu} = G_Q \Delta T \]

for small temperature difference

Linear geometry (Sample B)

\[ P^B_{\nu} / P^A_{\nu} = \frac{2}{5} \left( \frac{k_B TRC}{\hbar} \right)^2 \]

\[ \simeq 10^{-3} \]

in that experiment
Results in the two sample geometries

Heat transported by residual quasiparticles at $T > 0.3$ K and by photons (in the loop sample) at $T < 0.3$ K
Photon transport over a macroscopic distance

Quantum limit of heat flow across a single electronic channel

The Otto cycle
Quantum heat engines (quantum Otto refrigerator)

Niskanen, Nakamura, Pekola, PRB 76, 174523 (2007)

\[ \omega_{LC,1} = \frac{\Delta E_0}{\hbar} \]
\[ \omega_{LC,2} = \frac{\Delta E_A}{\hbar} \]

B. Karimi and JP, 2016
Properties of the qubit refrigerator

...in different frequency regimes

\[ P_2 = \frac{\hbar \omega_2}{2} \left[ \tanh\left( \frac{\beta_2 \hbar \omega_2}{2} \right) - \tanh\left( \frac{\beta_1 \hbar \omega_1}{2} \right) \right] f \]

B. Karimi and JP, in preparation
Quantum heat switch

Heat current between the two resistors under static conditions

\[
P_{12} = 2\Delta^2 g_1 g_2 \left( \frac{E_0^2}{\hbar} \right) \frac{(1 - e^{-\beta_1 \hbar \omega})^{-1}(e^{\beta_2 \hbar \omega} - 1)^{-1} - (1 - e^{-\beta_2 \hbar \omega})^{-1}(e^{\beta_1 \hbar \omega} - 1)^{-1}}{g_1[1 + Q_2^2\left( \frac{\omega}{\omega_{LC,2}} - \frac{\omega_{LC,2}}{\omega} \right)^2]^2 \coth\left( \frac{\beta_1 \hbar \omega}{2} \right) + g_2[1 + Q_1^2\left( \frac{\omega}{\omega_{LC,1}} - \frac{\omega_{LC,1}}{\omega} \right)^2]^2 \coth\left( \frac{\beta_2 \hbar \omega}{2} \right)}
\]
Fast NIS thermometry for studies of time-dependent phenomena

Read-out at 600 MHz of a NIS junction, 10 MHz bandwidth

S. Gasparinetti et al., Phys. Rev. Applied 3, 014007 (2015);
Proof of the concept: Schmidt et al., 2003
Fast NIS thermometry

Transmission read-out at 600 MHz of a NIS junction

- $T_{\text{bulk}} = 20 \text{ mK}$
- $V_{H_{\text{pp}}} = 0.01\ldots 0.2 \text{ V}$

Graph showing $P(\text{dB})$ vs. $T_e(\text{K})$ and $V_{H_{\text{pp}}}$ vs. $t(\mu\text{s})$. 

Resonator and NIS at 1 $\mu$m resolution with heater and ground markings.
Heat capacity $C$

\[ C \frac{d\delta T}{dt} = -G_{th} \delta T \]

\[ \tau = \frac{C}{G_{th}} \]

$T_{bath}$

$C$ of copper films is anomalously high (x10)

Silver follows free-electron Fermi-gas model

\[ C = \left(\frac{\pi^2}{3}\right) N(0)k_B^2V T \]
Summary

Presented:

measurement principles and techniques to investigate heat transport in nanostructures

basic heat transport mechanisms of quantum nanostructures

recent experiments and future plans
PICO group from the left: Minna Günes, Robab Najafi Jabdaraghi, Klaara Viisanen, Shilpi Singh, Jesse Muhojoki, Anna Feshchenko, Elsa Mannila, Mattijs Mientki, Jukka Pekola, Ville Maisi, Joonas Peltonen, Bivas Dutta, Matthias Meschke, Libin Wang, Antti Jokiluoma, Alberto Ronzani, Dmitri Golubev, Jorden Senior. Separate photos: Olli-Pentti Saira, Bayan Karimi