

# *Intensive course on*

## Paracontrolled distributions and applications to singular SPDEs

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(More details in <http://wiki.helsinki.fi/display/andy/Courses>)

One of the main objectives of stochastic analysis is the study of randomly evolving systems, whose dynamics are usually described in terms of stochastic differential equations. However, some of the stochastic partial differential equations of importance in mathematical physics are classically ill-posed; for example the KPZ equation, the dynamic  $\phi_d^4$  model, or the parabolic Anderson model. Due to the extremely irregular nature of the white noise the nonlinear operations appearing in their formulation are not easy to interpret rigorously. A major breakthrough towards these problems was recently achieved by Hairer with the help of his regularity structures.

I will present an alternative approach which was developed in parallel to regularity structures and is equivalent for many problems, but which is based on quite different tools. Our approach combines ideas from the theory of controlled rough paths with paraproducts, a notion from functional analysis which turns out to be perfectly suited for extending these ideas from functions of a one-dimensional time-parameter to functions of a multi-dimensional space parameter. In the lectures we will concentrate on a simple model example: homogenization of a diffusion in a highly oscillatory random environment. Here all the essential difficulties of singular SPDEs are present but many technicalities simplify compared to more complex examples.

In the first lecture I will introduce the basic philosophy and discuss some of the examples that can be studied using paracontrolled distributions, without going into technical details. We then start by having a closer look at the homogenization problem and develop the required tools (Littlewood-Paley decomposition, Besov spaces, Bony's paraproduct, Schauder estimates, paracontrolled analysis) on the way, as we need them. Most of the lectures will be based on the joint notes with Massimiliano Gubinelli [GP15a].

## References

- [CC13] R. Catellier and K. Chouk. Paracontrolled Distributions and the 3-dimensional Stochastic Quantization Equation. *arXiv:1310.6869*, 2013.
- [GIP15] M. Gubinelli, P. Imkeller and N. Perkowski. Paracontrolled distributions and singular PDEs. *Forum of Mathematics, Pi*, 29:1–89, 2015.
- [GP15a] M. Gubinelli and N. Perkowski. Lectures on singular stochastic PDEs. *Ensaos Matemáticos*, 29:1–89, 2015.
- [GP15b] M. Gubinelli and N. Perkowski. KPZ reloaded. *arXiv:1508.03877*, 2015.
- [MW16] J.-C. Mourrat and H. Weber. Global well-posedness of the dynamic  $\phi_3^4$  model on the torus. *arXiv:1601.01234*, 2015.