Demographic fluctuations in a population of anomalously diffusing individuals

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Galton-Watson process

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- Extinction in time $t_{ext} \sim N_0\Gamma^{-1}$.
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- Summing up to time $t$:  
  \[
  \sigma^2_N(t) = 2N_0 \Gamma t
  \]
Including some spatial structure

Divide the domain in cells and distribute bugs uniformly in them.

\[ n(x, t) \]

\[ L \]

\[ t=0 \]

Wait some time...
Including some spatial structure

Taller towers balance on the average increasingly wider gaps where the bugs have gone extinct.
Brownian bugs

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  - Constant mean density if bugs are initially uniformly distributed:
    \[ \rho_1(x, t) \equiv \langle n(x, t) \rangle = n_0 \]
  - Clustering behavior in \( D \leq 2 \):
    \[ \sigma_{n(x,t)}^2 \propto t^{1/2} \quad (D = 1); \quad \sigma_{n(x,t)}^2 \propto \ln t \quad (D = 2). \]
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  - Correlation length of fluctuations determined by Brownian motion scale \( \lambda(t) \sim (\kappa t)^{1/2} \).
Again simple mechanism

- Smearing effect by diffusion

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Again simple mechanism

- Smearing effect by diffusion
  \[ \sigma_n^2(x,t) \sim n_0(\kappa t)(\lambda(t))^{-D} \]
- \( \sigma^2 \propto t \) maximal growth in Galton-Watson case.
- Recall \( \lambda(t) \propto t^{1/2} \), that precludes clustering for \( D = 3 \).
We wander whether we could extend the approach to more general (anomalous) diffusion processes.
Practical relevance

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- Migration in the presence of long jumps (Lévy flights $\Rightarrow$ superdiffusion).
- Ageing in mutation phenomena (migration in genotype space; selection as a form of clustering in genotype space).
An important issue

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does not define uniquely the diffusion process.

- Four classes could roughly be identified:
  - Gaussian processes, such as the fractional Brownian motion; individuals that migrate with a power-law correlated velocity (generalized Langevin equation).
  - Continuous time random walk (CTRW): discrete jumps are separated by waiting time characterized by a distribution with heavy tails.
  - Migration in a spatial assembly of random traps
  - Migration by Lévy flights (not properly a diffusion process).
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Prescription problem

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- Fluctuations are accounted for by the connected family trees in (a).

- The disconnected trees in case (b) contribute \( n_0^2 \) to the correlation

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- It is not clear whether the bug generated in $(z, \tau)$ should preserve memory of the trajectory followed by its ancestors or not.
A closer look at connected trees

\[
\rho_{2c}(x_1, x_2; t) = 2\Gamma \int_0^t d\tau \int dz \int dw \, \rho_1(x_1, t|z, \tau; w, 0) \\
\times \rho_1(x_2, t|z, \tau; w, 0) \rho_1(z, \tau|w, 0) \rho_1(w, 0).
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As already stated, to calculate the connected correlation \(\rho_{2c}(x_1, x_2; t)\), we must consider family trees like the one beside.

As diffusion is non-Markovian, the conditioning on \((w, 0)\) in the \(\rho_1(x_{1,2}, t|z, \tau; w, 0)\) entering the equation for \(\rho_{2c}\) is not automatically irrelevant.
The case with no memory

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- Its trajectory will be the realization of an anomalous diffusion path: 
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The effect on clustering

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- Indeed, in the case of a CTRW:

- Growth of $\sigma_n^2$ for $D = 1$ and $H = 0.25$ (subdiffusion). Case (a) is the one with no memory. Insert: scaling of the correlation length $\lambda(t)$. The slopes $t^{1/2}$ are shown for comparison.
The effect on clustering

- We expect to recover the same scaling as in the Brownian bug case.
- Again, in the case of a CTRW:

\[ \tilde{C}(0, t, \tilde{t}) \]

- Growth of \( \sigma_n^2 \) for \( D = 1 \) and \( H = 0.75 \) (superdiffusion). The steeper lines come from including memory in a CTRW (\( s \)) and working with traps (\( d \)) (tough trouble; wait and see).
Recall the equation for $\rho_{2c}$:

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\rho_{2c}(x_1, x_2; t) = 2\Gamma \int_0^t d\tau \int dz \int dw \, \rho_1(x_1, t|z, \tau; w, 0) \\
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Assume $\rho_1(x, t|.)$ as in case with no demography.
Memory + Gaussian diffusion

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Carrying out the integrals we get after simple algebra

$$
\rho_{2c}(x_1, x_2; t) = \frac{\Gamma n_0}{2\sqrt{\pi}} \int_0^t \frac{d\tau}{\sigma(t; \tau)} \exp \left[ - \frac{(x_1 - x_2)^2}{4\sigma^2(t, \tau)} \right].
$$

$$
\sigma^2(t, \tau) = \sigma^2(t) - \frac{\langle y(t)y(\tau) \rangle^2}{\sigma^2(\tau)};
$$

$$
\sigma^2(t) = \kappa_H |t|^{2H}; \quad y(t) = x(t) - x(0).
$$
Anomalous scaling

- If $t \gg \tau$, $\sigma(t, \tau) \rightarrow \sigma(t)$; no singularity for $\tau \rightarrow t$. 
Anomalous scaling

- If $t \gg \tau$, $\sigma(t, \tau) \to \sigma(t)$; no singularity for $\tau \to t$.

- Power counting gives us $\rho_{2c}(x, x; t) = C \Gamma n_0 \kappa_H^{1/2} t^{1-H}$.
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Fluctuation build-up for a Gaussian superdiffusive process ($H = 0.75$; heavy line). The line $t^{1-H}$ is shown for comparison. Thin line obtained from Lévy flights (wait and see).
The other kingdom

Non-Gaussian processes

- CTRW-bugs: two throws of dice at each time: one to decide how long to wait; one to decide where to go. Waiting-time PDF with heavy tail to produce anomalous diffusion. Subdiffusion easy to get.

- Lévy-bugs: the same as Brownian bugs; the jump PDF is now heavy-tailed.
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Notice that Lévy bugs are Markovian; hence, they do not have a prescription problem.
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- Anomalous scaling is produced in the two cases by events in the heavy tails. Migration by CTRW is dominated by the longer waiting times; migration by Lévy flights is dominated by the longer jumps (only superdiffusion possible in this case).
Lévy flights

Markovian dynamics $\Rightarrow$ evolution equation for $\rho_{2c}$ local in time

$$\rho_{2c}(x_1, x_2; t + \Delta t) = \int dy_1 \int dy_2 \rho_1(x_1, t + \Delta t|y_1, t) \times \rho_1(x_2, t + \Delta t|y_2, t) \rho_{2c}(y_1, y_2; t) + 2\Gamma n_0 \Delta t \delta(x_1 - x_2).$$
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- Power-law tails: $\rho_1(x, \Delta t | 0, 0) \propto |x|^{-1-\beta}$, $0 < \beta < 1$

$\Rightarrow \rho_{1k}(\Delta t) \simeq 1 - \alpha |k|^\beta \Delta t$. Notice $\alpha^{2/\beta} \sim (\Delta x)^2 / (\Delta t)^{1/\beta}$, diffusivity-like quantity.
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**Equation for generating function**

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Forced “fractional” diffusion equation.
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\rho_{2c}(x_1, x_2; t + \Delta t) = \int \! \! \int \, d y_1 \, d y_2 \, \rho_1(x_1, t + \Delta t | y_1, t) \\
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Forced “fractional” diffusion equation.
The Lévy flights are characterized by infinite second moment $\langle |x(t) - x(0)|^2 \rangle = \infty$ also for a single jump. Nevertheless, $\rho_{1k}(\Delta t) \simeq 1 - \alpha |k|^{\beta} \Delta t$ identifies a characteristic length $y(t) \propto t^{1/\beta}$ that gives the scale of the clusters at time $t$. 
Anomalous “diffusion” again

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Recall the behavior \( (1/\beta = 0.75; \text{thin line}; \text{the heavy line described Gaussian superdiffusion}) \).
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The fluctuation level in a given cell is obtained summing the contributions from the various colonies that form there in the time $t$. 

**CTRW with memory**
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Since the average number of individuals in a colony is constant \((= 1)\) and the average number of individuals in the cell must also be constant, the colonies that form must be inversely proportional to their lifetime, and therefore:

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\sigma_{cell}^2 \sim \int_0^t d\tau \frac{d\text{Colonies}(\tau)}{d\tau} \times \sigma_{N_{colony}}^2(\tau) \sim t
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This is the same behavior of bugs that do not migrate!
Bugs in random traps

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- They seem to be VERY social bugs; actually, bugs arriving in the same trap, form colonies that disperse at the same time, while “simply social” colonies sharing a same location in space, do not share an identical dispersal time.
- However, “simply” social bugs are already characterized by a maximal level of fluctuation ($\sigma_n^2 \propto t$) and random traps make bugs to disperse even less than in the simple social case.
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However, “simply” social bugs are already characterized by a maximal level of fluctuation ($\sigma_n^2 \propto t$) and random traps make bugs to disperse even less than in the simple social case.

We expect the fluctuation growth in the case of random traps to remain maximal.
Numerical results

Growth of $C(0; t, t)$ in $D = 1$ and subdiffusive regime. Antisocial bugs ($a$), social bugs ($s$) and bugs in random traps ($d$). Insert: scaling of $\lambda(t)$ in the “antisocial case”.
Numerical results

Growth of $C(0; t, t)$ in $D = 1$ and superdiffusive regime. Antisocial bugs ($a$), social bugs ($s$) and bugs in random traps ($d$). Insert: scaling of $\lambda(t)$ in the “antisocial case”.
Numerical results

Sequence of snapshots of a population in a 2D field of traps. A small diffusivity is added to mimic the effect of small scale individual motion.
Numerical results

Growth of $C(0; t, t)$ in $D = 2$ (a) and $D = 1$ (b) for different mean numbers of bugs per trap (values range from 1 to 200).
Gaussian anomalous diffusion (FBM, individuals moving with velocity that is solution of a GLE, and others): if offsprings share memory of the trajectories with their parents, demographic fluctuations will scale anomalously. Otherwise, the case of Brownian bugs is recovered.
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- CTRW: in the absence of memory transfer between generations, the case of Brownian bugs is recovered. If the offsprings share their escape time with their parents, the dynamics falls back on that of a Galton-Watson process.
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- A spatial assembly of random traps: the dynamics falls back again on that of a Galton-Watson process.
Conclusion

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Galton-Watson like behaviors recovered in the case of random traps and CTRW with memory. Open question: what happens in the case of the traps, when they are distributed with respect to size and waiting times?
Conclusion

- In the case of non-Markovian processes, memory transfer among generation is the crucial ingredient to produce departure from Brownian bug behaviors at the population level.

- An effective alternative is provided by Lévy flights.

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