Mathematics Meets Physics

on the occasion of
Antti Kupiainen's 60th birthday

June 24-27, 2014
Helsinki, Finland
Schedule

Tuesday, June 24

09:00 – 10:15  registration + coffee
10:15 – 10:35  opening *
10:35 – 11:25  Krzysztof Gawędzki: Topological insulators and Wess-Zumino action
11:25 – 11:40  break
11:40 – 12:30  Denis Bernard: Quantum Jumps, Quantum Noise, and (Quantum) Applications
12:30 – 14:15  lunch break
14:15 – 15:05  Wendelin Werner: Critical models within critical models
15:05 – 15:45  coffee break
15:45 – 16:35  Kurt Johansson: The two-periodic Aztec diamond
16:40 – 17:30  Kari Astala: Rotational multifractal spectra for biLipschitz deformations
18:00 – 20:00  City of Helsinki reception and a music performance **

* Talks and opening: room P II (Porthania building, Yliopistonkatu 3)

** City of Helsinki Reception: at the City Hall, Pohjoisesplanadi 11-13. The event starts with a music performance; we kindly ask you to arrive by 18:00.
Wednesday, June 25

09:15 – 10:05 Ludwig Faddeev: *One spectral problem, from CFT*

10:05 – 10:45 coffee break

10:45 – 11:35 Giovanni Gallavotti: *Friction, reversibility, fluctuations in nonequilibrium and chaotic hypothesis*

11:40 – 12:30 Sergei Kuksin: *Weakly non-linear resonant hamiltonian PDEs and the problem of wave turbulence*

12:30 – 14:15 lunch break

14:15 – 15:05 Jean Bourgain: *On the density of states of Schrodinger operators*

15:05 – 15:45 coffee break

15:45 – 16:35 Michael Benedicks: *Parameter selection in one-dimensional maps revisited*

16:40 – 17:30 Viviane Baladi: *Breakdown of linear response in the presence of bifurcations*

17:45 – 18:15 Session on human rights of scientists, organized by Joel L. Lebowitz*

* Human rights session: room P II
Thursday, June 26

09:15 – 10:05  Joel Lebowitz: *Central Limit Theorems and Lee-Yang Zeros*

10:05 – 10:45  coffee break

10:45 – 11:35  Herbert Spohn: *Anharmonic chains and nonlinear fluctuating hydrodynamics in one dimension*

11:40 – 12:30  Stanislav Smirnov: *Scaling limit of the Ising model, loop ensembles and branching trees*

12:30 – 14:15  lunch break

14:15 – 15:05  Cédric Villani: *Long-time behavior of classical mechanical systems — from planets to plasmas to fluids*

15:05 – 15:45  coffee break

15:45 – 16:35  Michael Aizenman: *Resonant Delocalization on the Complete Graph*

16:40 – 17:30  Alan Sokal: *Some wonderful conjectures (but very few theorems) at the boundary between analysis, combinatorics and probability*

19:00 –  Conference dinner *

* The **conference dinner** will be held at **Restaurant Saari**, located on Sirpalesaari Island near the southern tip of Helsinki. This event is for those who have registered for it in advance. **Invitation cards** and separate instruction sheets on how to arrive there have been provided at registration. *We kindly ask you to bring the invitation cards with you.*
Friday, June 27

09:15 – 10:05  Jürg Fröhlich: TBA
10:05 – 10:45  coffee break
10:45 – 11:35  Giovanni Felder: Quantum integrable systems and hyperplane complements
11:40 – 12:30  Jean Bricmont: What did Bell really prove?
12:30 – 14:15  lunch break
14:15 – 15:05  Thomas Spencer: Statistical Mechanics arising from Random Matrix Theory
15:05 – 15:45  coffee break
15:45 – 16:35  Jean-Pierre Eckmann: Antti


**Titles and abstracts**

**Michael Aizenman**

Princeton University

*Resonant Delocalization on the Complete Graph*

Resonant delocalization plays a role in the formation of bands of semi-delocalized eigenstates for Schrödinger operators with disorder. The talk will focus on the way this mechanism appears in case of the random Schrödinger operator on the complete graph. The operator’s spectrum includes in addition to the localized eigenfunction also bands of semi-delocalized eigenstates, which emerge through hybridization from resonating local quasi-modes. In analyzing the latter it is of help to employ some general properties of the scaling limits of random functions in the Pick class. We find that in energy windows where semidelocalized eigenfunctions occur the rescaled spectrum resembles that of the Seba process, which would be defined in the talk. The eigenfunctions in such regimes are delocalized in the $L^1$ sense, though not in $L^2$. The results are found to be in agreement with a heuristic condition for the emergence of resonant delocalization in terms of the tunneling amplitude among quasi-modes.

Based on the joint works:


**Kari Astala**

University of Helsinki

*Rotational multifractal spectra for biLipschitz deformations*

**Viviane Baladi**

CNRS and ENS, Paris

*Breakdown of linear response in the presence of bifurcations*

(Joint work with: M. Benedicks and D. Schnellmann)

Many interesting dynamical systems possess a unique Sinai-Ruelle-Bowen ("physical") measure, which behaves well with respect to Lebesgue measure. Given a smooth one-parameter family of dynamical systems $f_t$, it is natural to ask whether the SRB measure depends smoothly on the parameter $t$. If the $f_t$ are smooth hyperbolic diffeomorphisms (which are structurally stable), the SRB measure depends differentiably on the parameter $t$, and its derivative is given by a "linear response" formula (Ruelle, 1997). When bifurcations are present and structural stability does not hold, linear response may break down. This was first observed for piecewise expanding interval maps, where linear response holds for tangential families, but where a modulus of continuity $t \log t$ may be attained for transversal families (Baladi-Smania, 2008). The case of smooth unimodal maps is much more delicate. Ruelle (Misiurewicz case, 2009) and Baladi-Smania (slow recurrence case, 2012) obtained linear response for fully tangential families (confined within a topological class). We shall present our new results with Benedicks and Schnellmann on the transversal smooth unimodal case, including
the quadratic family (for which we obtain Holder upper and lower bounds, in the sense of Whitney, along suitable classes of parameters).

Michael Benedicks
KTH Royal Institute of Technology, Stockholm

Parameter selection in one-dimensional maps revisited

There are several ways to construct abundance (positive measure) of parameters with absolutely continuous invariant measures for one-dimensional maps. The first is the famous construction due to Jakobson. Another is due to Carleson and myself in the case of the quadratic family. I will discuss this later proof in view of its basic ingredients, and which generalizations may be possible in the one-dimensional real and complex case.

Denis Bernard
CNRS and ENS, Paris

Quantum Jumps, Quantum Noise, and (Quantum) Applications

Jean Bourgain
IAS, Princeton

On the density of states of Schrodinger operators

We will discuss several problems related to the regularity of the density of states of Schrodinger operators (SO). The first is the continuity problem for SO’s on $\mathbb{R}^d$ with bounded potential. The second is the smoothness issue for the 1D Anderson Bernoulli model at small disorder. The underlying mathematics has to do with quantitative Carleman inequalities and group expansion.

Jean Bricmont
UCL, Louvain-la-Neuve

What did Bell really prove?

The goal of this talk is to give a pedagogical introduction to Bell’s theorem and its implication for our view of the physical world, in particular on the issue of locality. We will also discuss several misunderstandings of Bell’s result, as well as of Einstein’s views on quantum mechanics.

Wojciech De Roeck
K.U. Leuven

Irreversibility in Hamiltonian dynamics

Some of Antti’s work concerns the transition from microscopic to macroscopic laws of physics, and more particularly the emergence of irreversibility. I played in part in one variation on this theme,
connected to quantum mechanics. I will try in this talk to give a hint of these topics for an audience that probably is not familiar with quantum mechanics. A guiding question could be: Why do physical systems approach equilibrium and why is this hard to prove?

Jean-Pierre Eckmann  
University of Geneva  
Antti

Ludwig Faddeev  
Steklov Mathematical Institute, St.Petersburg  
*One spectral problem, from CFT*  
Let $u$ and $v$ be Weyl pair $uv = q^2 vu$, realized as selfadjoint operators in $L_2(R)$. The operator $H = u + u/\text{inv} + v$ appears in many places from QCFt to quantum Teihmuller theory. I consider a spectral problem for $H$, following the joint work with Leon Takhtajan.

Giovanni Felder  
ETH Zurich  
*Quantum integrable systems and hyperplane complements*  
The Gaudin spin chains are a prototypical family of quantum integrable systems. In their most general form they may be defined as degree 1 maximal abelian subalgebras of Kohno’s holonomy Lie algebra of the complement of an arrangement of hyperplanes. I will discuss the occurrence of Gaudin subalgebras in representation theory and the geometry of the variety of Gaudin subalgebras. Based on joint work with L. Aguirre and A. Veselov and on Aguirre’s thesis.

Giovanni Gallavotti  
Universita di Roma 1  
*Friction, reversibility, fluctuations in nonequilibrium and chaotic hypothesis*  
I shall discuss possible extensions of the theory of ensembles and their equivalence to nonequilibrium statistical mechanics and chaotic evolutions by presenting some concrete examples.

Krzysztof Gawędzki  
ENS, Lyon  
*Topological insulators and Wess-Zumino action*  
I shall discuss a generalization of the Kane-Mele $Z_2$ invariant for topological insulators to the case of systems periodic in time and a resulting new interpretation of the stationary invariant in terms of $Z_2$-equivariant Wess-Zumino action functional. This could have been a joint work with Antti 25 years ago.

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June 24–27 2014, Helsinki
Kurt Johansson  
KTH Royal Institute of Technology, Stockholm  

The two-periodic Aztec diamond  

Random domino tilings of the region called the Aztec diamond exhibit interesting features and statistical properties related to random matrix theory. As a statistical mechanical model it can be thought of as a dimer model or as a certain random surface. The Aztec diamond with a two-periodic weighting exhibits all the three possible phases that can occur in these models usually called liquid, solid and gas. This model is considerably harder to analyze than the standard one-periodic model which only has two phases, liquid and solid. I will give an overview of some of the asymptotic results that have been obtained in ongoing joint work with Sunil Chhita. The analysis is based on the fact that we are able to obtain a useful double contour integral formula for the inverse Kasteleyn matrix of the model and it thus gives an interesting exactly solvable statistical mechanical model.

Peter Jones  
Yale University  

TBA  

Sergei Kuksin  
CNRS and Paris 7  

Weakly non-linear resonant hamiltonian PDEs and the problem of wave turbulence  

I will discuss long-time behaviour of small oscillations in a non-linear Shroedinger equation on a torus, perturbed by random force and linear dissipation. The equation is scaled in such a way that its solutions are small, but their limiting dynamics is non-trivial. The limiting behaviour turns out to be described by another damped/driven Hamiltonian PDE, where the new Hamiltonian is constructed out of the resonant terms of the original one. Next I will discuss behaviour of the new system under the limit “space-period goes to infinity”. Using heuristic approximation, commonly used in the weak turbulence, I will derive for the second limit a KZ-type kinetic equation which leads to KZ energy spectra. I will present numerics which support the predictions concerning the second limit.

Joel Lebowitz  
Rutgers University  

Central Limit Theorems and Lee-Yang Zeros  

Stanislav Smirnov  
University of Geneva and St. Petersburg State University  

Scaling limit of the Ising model, loop ensembles and branching trees  

We will discuss our attempts with Antti Kemppainen to describe geometrically the scaling limit of the Ising model at criticality. Considering the random cluster representation, we can picture it
as a collection of nested self-touching loops - perimeters of clusters. Starting from some point \( a \) there is a canonical way to reach any other point \( z \) jumping from one perimeter to another. Taking such roots for all points \( z \), we arrive at a nice tree-like structure. It appears to have a conformally invariant scaling limit, with SLEs with drifts (namely, SLE\((16/3, -2/3)\)) naturally arising as limits of individual branches.

Alan Sokal
New York University and University College London

Some wonderful conjectures (but very few theorems) at the boundary between analysis, combinatorics and probability

Many problems in combinatorics, statistical mechanics, number theory and analysis give rise to power series (whether formal or convergent) of the form

\[
f(x, y) = \sum_{n=0}^{\infty} a_n(y)x^n
\]

where \( \{a_n(y)\} \) are formal power series or analytic functions satisfying \( a_n(0) \neq 0 \) for \( n = 0, 1 \) and \( a_n(0) = 0 \) for \( n > 1 \). Furthermore, an important role is played in some of these problems by the roots \( x_k(y) \) of \( f(x, y) \) — especially the “leading root” \( x_0(y) \), i.e. the root that is of order \( y^0 \) when \( y \to 0 \). Among the interesting series \( f(x, y) \) of this type are the “partial theta function”

\[
\Theta_0(x, y) = \sum_{n=0}^{\infty} x^n y^{n(n-1)/2},
\]

which arises in the theory of \( q \)-series, and the “deformed exponential function”

\[
F(x, y) = \sum_{n=0}^{\infty} (x^n/n!) y^{n(n-1)/2},
\]

which arises in the enumeration of connected graphs and in the partition function of the Potts model on the complete graphs. These two functions can also be embedded in natural hypergeometric and \( q \)-hypergeometric families.

In this talk I will describe recent (and mostly unpublished) work concerning these problems — work that lies on the boundary between analysis, combinatorics and probability. In addition to explaining my (very few) theorems, I will also describe some amazing conjectures that I have verified numerically to high order but have not yet succeeded in proving. My hope is that one of you will succeed where I have not!

Further information is available at [http://www.maths.qmul.ac.uk/~pjc/csgnotes/sokal/](http://www.maths.qmul.ac.uk/~pjc/csgnotes/sokal/)

Thomas Spencer
IAS, Princeton

Statistical Mechanics arising from Random Matrix Theory

I will survey some theorems and conjectures about statistical mechanics models with non-compact symmetries which arise from random matrix models. A special case is related to a history dependent “edge reinforced random walk”.

June 24–27 2014, Helsinki
Herbert Spohn  
TU Munich  

*Anharmonic chains and nonlinear fluctuating hydrodynamics in one dimension*

There is convincing numerical evidence that the equilibrium time correlations of anharmonic chains are well captured by a nonlinear extension of fluctuating hydrodynamics. We explain the theory and discuss the universality classes.

Cédric Villani  
IHP, Paris  

*Long-time behavior of classical mechanical systems — from planets to plasmas to fluids*

Wendelin Werner  
ETH Zürich  

*Critical models within critical models*

I will discuss some answers to the following type of questions (building on joint ongoing work with Jason Miller and Scott Sheffield): Can one make sense of critical percolation or other critical models, with the fractal clusters of critical planar models? If so, what happens to its conformal invariance?